This is an electronic appendix to the paper by Stephens *et al*. 2004 Impulsiveness without discounting: the ecological rationality hypothesis. *Proc. R. Soc. B* **271**, 2459-2465. (doi: 10.1098/rspb.2004.2871)

Electronic appendices are refereed with the text. However, no attempt is made to impose a uniform editorial style on the electronic appendices.

Electronic Appendix A

1. More Cases for Study 1.

The printed article shows a single "characteristic" plot for our first study (fig. 3a), with undiscounted fitness. While this plot illustrates our main finding (impulsiveness in the patch situation, but not in self-control situation), we observed a wide range of behavior in the self-control situation, especially with low discrimination accuracy (*D* small). Figure 5 shows five cases that illustrate the effect of the mean and variance of travel time on the self-control rule. Each panel shows calculated ρ values as a function of the discrimination accuracy parameter for a different set of τ values. The upper dotted curve shows the optimal ρ_t for the self-control situation. The solid line shows ρ_t for the patch situation, and the dashed line shows ρ_A for the patch situation. Recall that ρ_A is irrelevant in the self-control situation because neither option provides a second food delivery, so we do not show it. The first column shows a sequence of results for the case where there is a single ITI in the choice environment; illustrating the effects of changing the magnitude of τ. The second column shows the effect of variance in τ, by showing two cases where τ is an equal mix to two values with mean value 3; the smallest value in this mix is always the same as the single value in the on graph to the left. Notice that columns are similar suggesting that the lowest value in a mix of τ's controls the optimal rule. The two panels in last row are identical because the only mix of two values that includes 3 and has mean 3 is “3 and 3”.

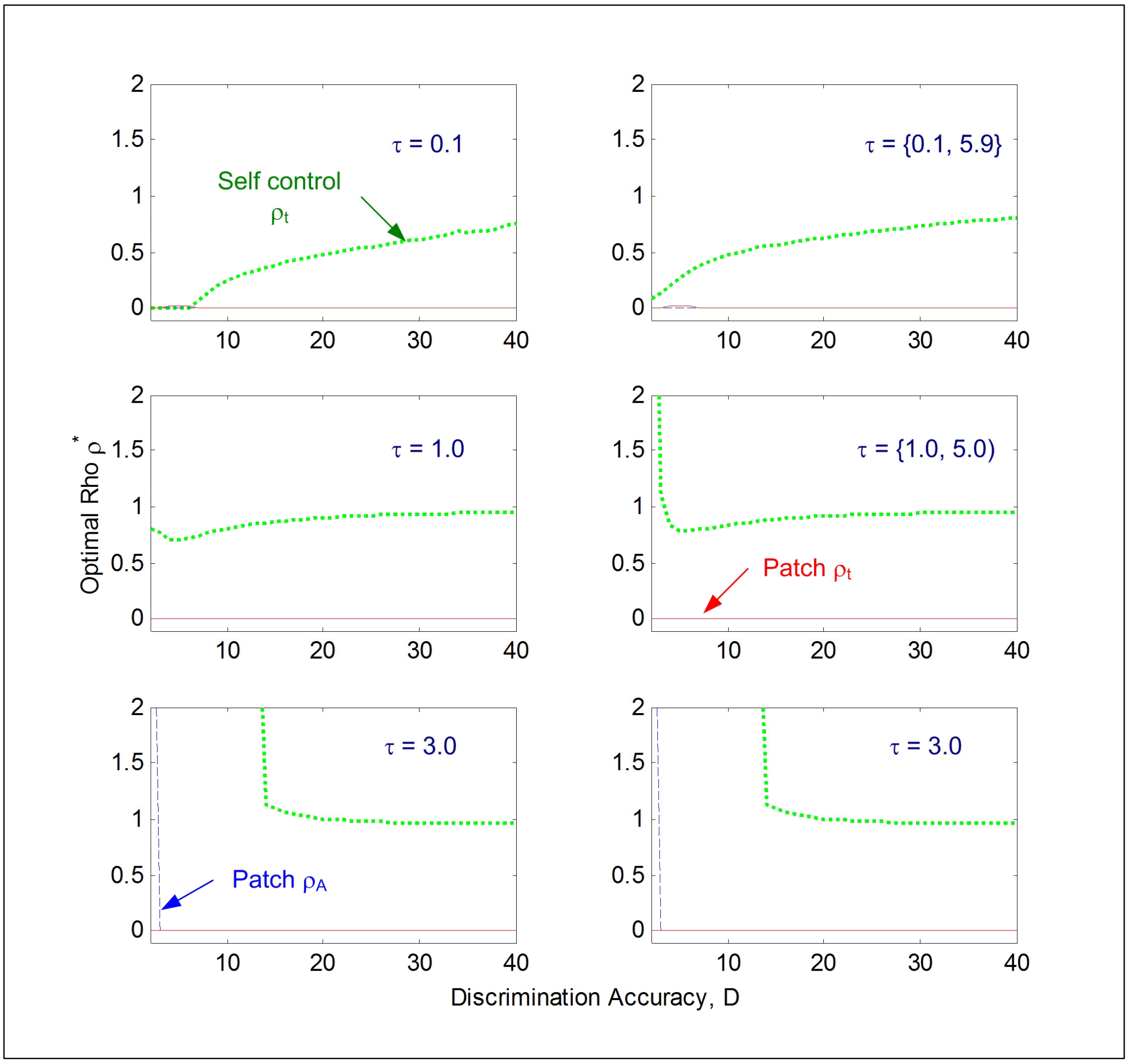

Figure 5

2. Mathematical Appendices

(a) Discounted Fitness

Here we derive equation (8). Consider the standardized choices of figure 2. If the subject chooses the unprimed option with probability p and food is discounted at a rate λ, then the expected discounted value of a single choice is

$$s_1 = p(e^{-\lambda t_1} A_1 + e^{-\lambda(t_1+t_2)} A_2) + (1-p)(e^{-\lambda t_1'} A_1' + e^{-\lambda(t_1'+t_2')} A_2').$$

The value of a sequence of two choices is

$$s_2 = p(e^{-\lambda t_1} A_1 + e^{-\lambda(t_1+t_2)} A_2 + e^{-\lambda(t_1+t_2)} s_1) + (1-p)(e^{-\lambda t_1'} A_1' + e^{-\lambda(t_1'+t_2')} A_2' + e^{-\lambda(t_1'+t_2')} s_1).$$

We can readily generalize this logic. If s_i and s_{i-1} are the expected discounted values of a sequence of i and $i-1$ choices, respectively, then we can define a one-dimensional linear map f giving a general recursion for value, as follows:

$$s_i = f(s_{i-1}) = \left\{ p(e^{-\lambda t_1} A_1 + e^{-\lambda(t_1+t_2)} A_2) + (1-p)(e^{-\lambda t_1'} A_1' + e^{-\lambda(t_1'+t_2')} A_2') \right\} + \left\{ pe^{-\lambda(t_1+t_2)} + (1-p)e^{-\lambda(t_1'+t_2')} \right\} s_{i-1}$$

This map has a single equilibrium at

$$s^* = \frac{p(e^{-\lambda t_1} A_1 + e^{-\lambda(t_1+t_2)} A_2) + (1-p)(e^{-\lambda t_1'} A_1' + e^{-\lambda(t_1'+t_2')} A_2')}{1 - pe^{-\lambda(t_1+t_2)} - (1-p)e^{-\lambda(t_1'+t_2')}}$$

Because f is a *linear* map, the slope of this line (m) fully determines the stability of this equilibrium. If m>1, then the equilibrium is unstable, whereas if m<1, this equilibrium is *globally* stable (m=1 corresponds to neutral stability). Since,

$$\frac{df}{ds} = pe^{-\lambda(t_1+t_2)} + (1-p)e^{-\lambda(t_1'+t_2')},$$

and since λ>0, t_1+t_2>0 and $t_1'+t_2'$>0, we have $m = \frac{df}{ds} < 1$. Therefore, this system approaches the equilibrium s^*, despite starting conditions. Thus, as the number of choices approaches infinity (i.e., over the long term), the expected discounted value must converge to s^*, which justifies the use of equation (8).

(b) Optimal ρ's with perfect discrimination

This appendix shows the special relationships between optimal ρ_A and ρ_t values that arise when discrimination is perfect.

Case 1: No discounting

Perfect discrimination means that the discrimination function is 0 for 0<*x*<1 and 1 for *x*>1 (we assume that the discrimination function is ½ when *x*=1), that is, discrimination is a discontinuous step function. If *x*>1 and we have a patch environment then the following must hold:

$$\left(\frac{A_1}{A_1^{'} + \rho_A A_1}\right)\left(\frac{t_1^{'} + \rho_t t_1}{t_1}\right) > 1 \qquad \text{(A1)}$$

When *x*>1, the unprimed choice (leave option in the patch) is selected with certainty. This will be the correct choice when:

$$\frac{A_1}{t_1} > \frac{A_1^{'} + A_1}{t_1^{'} + t_1} \qquad \text{(A2)}$$

this can be rearranged to give:

$$A_1 t_1^{'} - A_1^{'} t_1 > 0 \qquad \text{(A3)}$$

We would like the weights in (A1) to be such that it is equivalent to inequality (A3). We can rearrange (A1) into the following inequality:

$$\frac{A_1 t_1^{'} - A_1^{'} t_1}{A_1 t_1} > \rho_A - \rho_t \qquad \text{(A4)}$$

Since $A_1 t_1$ is positive, (A4) is equivalent to (A3) when $\rho_A = \rho_t$. This argument can be repeated when 0<*x*<1 and when *x*=1 with the same results. As long as there is equal weight placed on future amounts and future weights, the organism will perform optimally.

Case 2: Discounting

Except for the case where x=1, the argument p in equation (8) will be 0 or 1 under perfect discrimination. Thus, the fitness in the patch environment with discounting at rate λ will be one of the following:

$$W_\theta(0) = \frac{A_1^{'} e^{-\lambda t_1^{'}} + A_1 e^{-\lambda\left(t_1^{'}+t_1\right)}}{1-e^{-\lambda\left(t_1^{'}+t_1\right)}} \tag{A5}$$

$$W_\theta(1) = \frac{A_1 e^{-\lambda t_1}}{1-e^{-\lambda t_1}} \tag{A6}$$

We would like x>1 when $W_\theta(1) > W_\theta(0)$, which can be rearranged to give the following inequality:

$$\frac{A_1'}{A_1} < \frac{e^{-\lambda t_1'}-1}{e^{-\lambda t_1}-1} \tag{A7}$$

And inequality (A1) can be rearranged to give:

$$\frac{A_1^{'}}{A_1} < \frac{t_1^{'}}{t_1} + \rho_t - \rho_A \tag{A8}$$

Inequality (A7) and (A8) are equivalent when:

$$\frac{t_1^{'}}{t_1} + \rho_t - \rho_A = \frac{e^{-\lambda t_1^{'}}-1}{e^{-\lambda t_1}-1} \tag{A9}$$

Consider a patch environment specified by a given set of times. We assume that the amounts vary in such a way that sometimes the unprimed choice is better and sometimes the primed choice is better. Under perfect discrimination, for a given value of λ, the organism acts optimally if its ρ values satisfy the following relationship:

$$\rho_A = \rho_t + \left\{\frac{t_1^{'}}{t_1} - \frac{e^{\lambda t_1^{'}}-1}{e^{\lambda t_1}-1}\right\} \tag{A10}$$

Any (ρ_A, ρ_t) combination on this line has equivalent fitness. Note that the limit of this relationship as $\lambda \to 0$ is simply $\rho_A = \rho_t$ (which was the optimal weighting with no discounting).